Experimental Comparison of State-of-the-Art Methods for Digital Optimum Filter Synthesis With Arbitrary Constraints and Noise

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***Abstract*—We present the experimental application and comparison of two methods for the synthesis of digital filters, which represent the state-of-the-art of optimum digital processing of shaped signals with arbitrary constraints in time and frequency domain, and any kind of stationary noise power spectral density. The methods are implemented in experimental measurement setups, and optimum filters are synthesized with regard to assigned constraints (e.g., finite duration, flat top, peaking time, zero area, etc.) and by taking into account the real environmental noise or disturbance present in the system, identified from datasets of simple signal experimental acquisitions. Implementation issues are detailed and basic design rules for digital signal processors based on these techniques are derived.**

***Index Terms*—DSP, energy resolution, high-resolution spectroscopy, HPGe detectors, LMS data analysis, optimum digital spectroscopy.**

I. Introduction

Many applications, e.g., nuclear electronics, biomedical, homeland security, etc., need accurate estimations of parameters such as energy and time of signals detected in noisy environments [1]–[3].

Optimum filter theory allows achieving parameter estimation with the best possible signal-to-noise ratio [4].

Optimum filter synthesis is addressed by taking into account the specific characteristics of both signal and noise, plus any other desired filter constraint (online techniques for radiation measurement often require filters with finite duration and flat top).

In spite of those benefits, optimum filters have not widely spread within all the applicative fields yet, mainly because of two reasons: 1) the intrinsic difficulty of implementing generically shaped filters by means of pure analog electronics; and 2) the uncertainty about the effectiveness of optimum digital filtering in comparison with suboptimal, but well-established, analog filters. It will be shown that it is indeed possible to synthesize filters that are supposed to be the optimal ones, but because they do not actually fit with the characteristics of the experimental environment, often result in even poorer performance than the traditional analog filters.

In order to prove the effectiveness of optimum filtering in dealing with energy estimation of experimental pulses, and to show the necessary steps required to synthesize filters very close to the optimum one, we compared three digital filters obtained from two different methods for optimum filter synthesis, and a well-established standard method for filter synthesis in terms of the resulting energy spectra resolution.

II. Synthesis Methods

A. Deconvolution Method

The basic idea behind this method is very simple and effective. The "moving window deconvolution" technique [5], which, among other benefits, perfectly suits online pulse processing, is indeed one of most popular and widely spread methods in the field of radiation measurement.

Given any analog pulse shaper before the ADC with its known-by-design number of singularities (usually only poles, at least to model the antialiasing filter) it is always possible to estimate the position of each singularity, either by means of direct inspection or using more refined algorithms [6]–[8].

In the case of direct sampling of the preamplifier output pulses, as in this experimental setup, signals should be exponentially decaying pulses, with their time constant set by the preamplifier R-C section, plus the additional "smoothing" effect of a couple of very-high-frequency poles, introduced, for example, by the distributed capacitance of the cable and/or the finite op-amp bandwidth of the preamplifier, which do not influence too much the decaying part of the pulse.

Based on this *a priori* assumption, all the deconvolution techniques consist of a proper digital filter that either reconstructs the original "delta-like" shape of the current signal collected by the charge preamplifier, or integrates that short "delta-like" signal to obtain a slightly larger rectangular shape. In both cases, the subsequent filters can be derived from optimum filter-synthesis tools [9], [10] or, for simple noise configurations, directly from closed-form calculations (for example, the well-known trapezoidal filters in the case of series noise only), but always under the fundamental assumption of a "delta-like" or "rectangular" shape at the filter input.

However, sometimes this is not the case (see Fig. 1), since many contributions impact on the final pulse shape, and make it difficult to predict *a priori* and/or calibrate the system whenever

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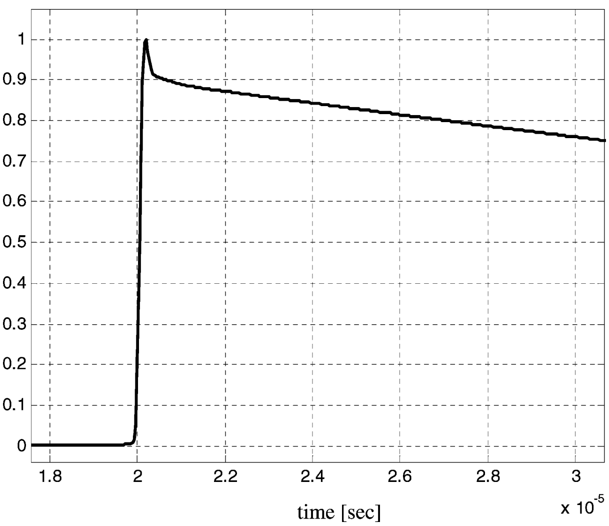

Fig. 1. Plot of the preamplifier output pulse for the experimental setup described in Section III.

a change is detected. Among those effects are the length and quality of cables from the preamplifier to the sampling ADC, the possible preamplifier peaking due to the lack of phase margin in the closed loop gain, etc.

Fig. 1 shows the preamplifier output pulse for the experimental setup described in Section III, which is clearly not purely exponentially decaying. This, in turn, slightly affects the resulting energy resolution, for the main reason that no real flat top is actually synthesized.

B. Fourier Method

This is a very general method, introduced by us in [9], to find the optimum weighting function (WF) for "delta-like" and quasi-"delta-like" pulses, in the presence of any kind of stationary noise. We demonstrated that the sought WF can be conveniently represented in the form of is a truncated Fourier sine series composed by symmetrical and antisymmetrical harmonics

$$\begin{aligned} \mathrm{WF}(t) &= \sum_n A_n \sin\left(n\frac{\pi}{T}t\right), \quad 0 < t < T \\ \mathrm{WF}(t) &\equiv 0, \qquad\qquad\qquad\qquad \text{elsewhere.} \end{aligned} \tag{1}$$

The optimum WF is, therefore, fully defined by the coefficients A_n, while the noise power spectrum is expressed in the frequency domain by means of the Laurent power series.

According to (1), the optimum WF is always limited in time to a single semiperiod of the fundamental harmonic to directly comply with the imposed duration T of the sought optimum WF, and is forced to zero anywhere else.

As a second step, a functional is built, which depends on the noise at the output of the filter and on all the desired constraints on the temporal shape of the WF (e.g., duration T, flat top, zero area, pulse shape, pulse occurrence time) expressed as Lagrange conditions. The optimum WF is finally obtained by minimizing that functional with respect to all the harmonics of the Fourier series up to a maximum one, which can be chosen by the user according to the desired accuracy.

C. Digital Penalized Least Mean Squares Method

The DPLMS algorithm for optimum filter synthesis [11] consists of three main steps.

First, the variance of the filter output noise is expressed as a function of the input noise, characterized from the experimental acquisition of a proper set of noise sequences, and as a function of the yet unknown digital filter. In this way, every input noise or disturbance, *a priori* known or unknown, is always taken into account, which allows synthesizing the optimum filter for the actual experimental condition.

Second, the input signal is sampled by the ADC, and this set of experimental acquisitions is averaged to obtain a noiseless reference input signal. All the desired constraints are then imposed on both the output signal and the digital filter, and a corresponding set of functionals is calculated, which indicates how well each constraint is satisfied as a function of the yet unknown digital filter.

Third, the calculated output noise variance and all the functionals calculated in the previous step are weighted and summed together, and that final functional is minimized with respect to all the coefficients of the digital filter.

Indeed, as input signal is a stochastic process, the most convenient way to impose a desired constraint on the WF consists of identifying a digital filter that gives an average output value equal to the desired one, while at the same time, reducing as much as possible the noise-related variance of the output signal. This approach is very general, and the most important constraints in the time and frequency domains can be imposed from this basic point of view. Since the sought digital filter X belongs to the class of filters with finite impulse response (FIR), its output signal simply consists of a weighted (x_j) sum of input signal samples (i_j).

III. Experimental Setup

A ^{60}Co source (10 000 events/s) has been put right on top of an HPGe coaxial detector (EG&G Ortec, Oak Ridge, TN), biased at 2 kV and connected to a spectroscopy preamplifier with 50 μs nominal decay time (EG&G Ortec, Oak Ridge, TN). The resulting electrical signals have been acquired using a digital oscilloscope (Agilent Infinium 54830D). Trigger level within the scope has been set around 5 mV above the baseline level, which realized a good compromise between the aim of also acquiring low-energy pulses, and the aim of avoiding too many noise-triggered events. Pulses have been acquired at a sampling rate of 25 MHz, 8-bit resolution, 1000 samples/pulse. The overall dataset ($10\,000 + 100\,000$ pulses) has been sent to a PC through a LAN connection using the Matlab Control Instrument Toolbox (The Mathworks, Natick, MA) and stored for subsequent processing. Although the chosen experimental setup (Fig. 2) is far away from the state-of-the-art DAQ systems for energy resolution, it has the fundamental advantage of being easily reproducible, as no custom digital electronics is required, while still allowing a full and direct comparison between the various digital filters under test.

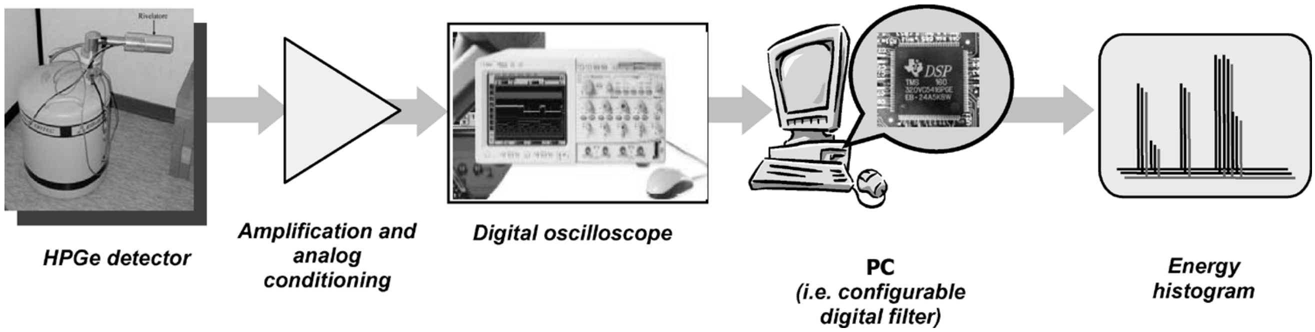

Fig. 2. Experimental setup used to evaluate the synthesized optimum filters for energy estimation of pulses.

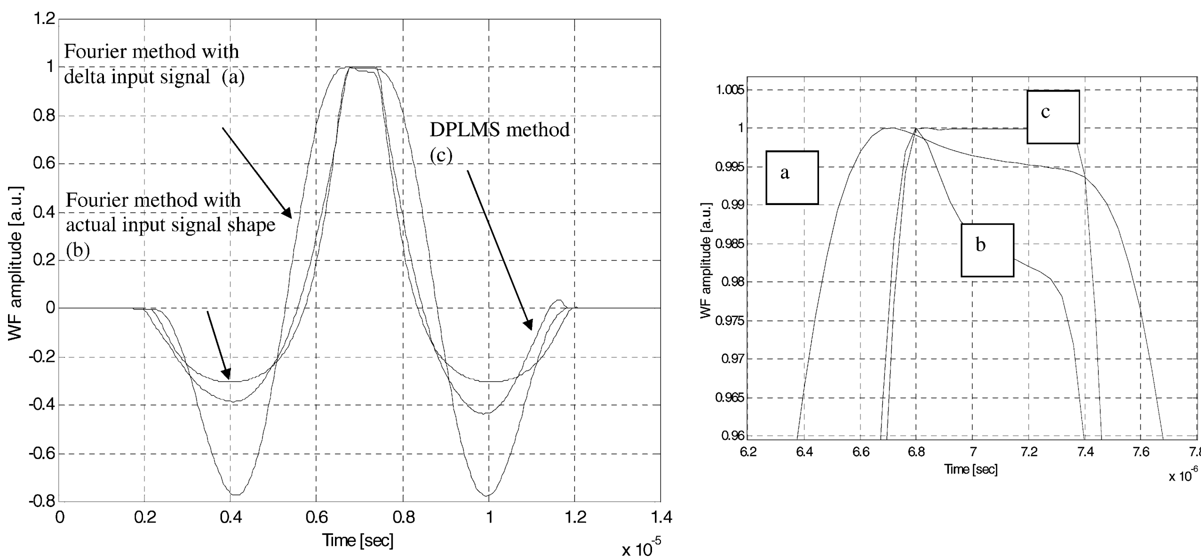

Fig. 3. Optimum WFs synthesized by the Fourier and the DPLMS methods. The insert plot shows a magnification of the flat-top zone of the waveforms. For testing the sensitivity on the signal shape, the Fourier method has been fed both with (a) an arbitrarily supposed delta signal and with (b) the real signal shape.

IV. Data Analysis

The first dataset of pulses (10 000 pulses) has been used to characterize the experimental setup. The noiseless preamplifier reference pulse and the system noise autocorrelation function (supposing only time-invariant noise contributions) have been estimated by averaging the dataset of pulses. The second and larger dataset of pulses (100 000 pulses) has been used to build the energy spectra by using the digital filters under test.

Four digital filters have been synthesized, all of which should provide WFs characterized by null area, finite duration (10 μs), and flat top (600 ns). The first filter does not take into account the actual setup noise, and simply implements the standard WF (trapezium + parabolas, optimized for voltage series noise only) available with most commercial digital spectroscopy processors. This is done by means of an equivalent moving window deconvolution algorithm [12], supposing the preamplifier output signal to be exponentially decaying, and estimating its time constant by a dedicated algorithm [7]. This filter is representative of standard signal processing, and is, thus, a good reference milestone for spectrum energy resolution. Both the second and the third filters have been synthesized with the Fourier algorithm [9], while for the fourth filter, we used the DPLMS [11] method (Fig. 3).

Fourier and DPLMS methods should theoretically synthesize the same digital filter. However, this is not the real case. While the DPLMS method simply works on the noise correlation function and the noiseless pulse shape of the digitized signals, and directly synthesizes the optimum FIR filter, the Fourier method needs more processing steps. Namely, the Fourier method must be provided with the estimated noise correlation function and the corresponding noiseless pulse prior to the preamplifier. Because this information cannot be gained by direct acquisition, several assumptions have to be made. Specifically, filter two has been synthesized by assuming that the pulse shape prior to the preamplifier is a “delta-like” signal, and by calculating the noise correlation function assuming a pure exponentially decaying preamplifier impulse response. Both hypotheses, however, are not completely correct, and the resulting filter has indeed the poorest performance among the four filters under evaluation. By providing the Fourier method with a more correct pulse-shape estimation (the one represented in Fig. 4), the corresponding filter indeed provides an energy resolution comparable to the reference one.

However, none of the WFs considered so far provides a good-quality flat top (see Fig. 3), because the model assumed for the preamplifier impulse response in this case is only partially correct (not a pure exponentially decaying pulse). On the contrary,

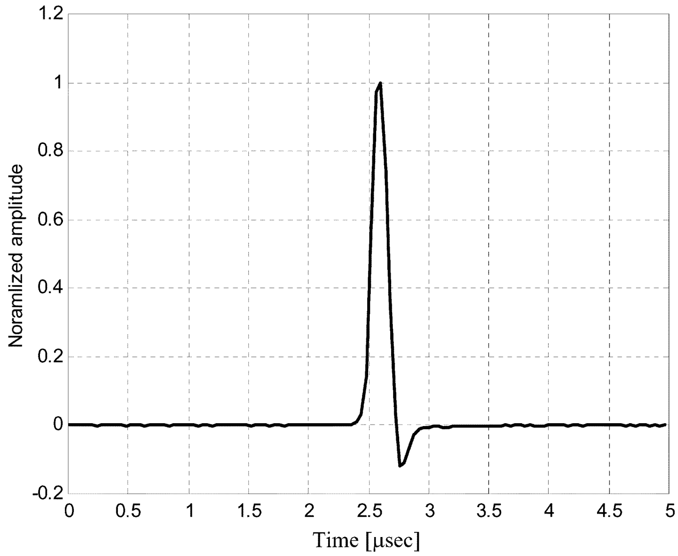

Fig. 4. Real signal shape referred at the input of the preamplifier.

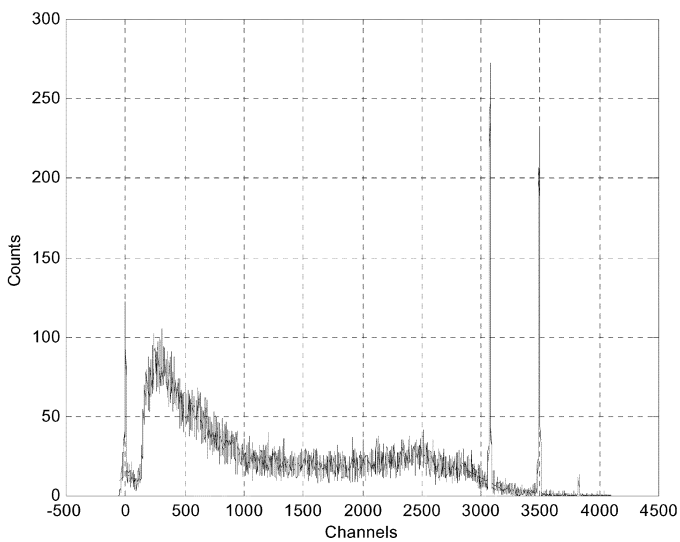

Fig. 5. Energy spectrum of the ^{60}Co source from the spectrometer that is initialized with the DPLMS method.

TABLE I

RESOLUTIONS OVER THE MAIN LINES OF A ^{60}Co SPECTRUM THROUGH CONSIDERED FILTER SYNTHESIS METHODS. ESTIMATION ERROR ALSO REPORTED (STANDARD DEVIATION OF EACH DISTRIBUTION), SUPPOSING ALL PEAKS SHOW GAUSSIAN DISTRIBUTION. THIS IS COMPLETELY TRUE ONLY FOR THE ELECTRONIC LINE PEAK, BECAUSE OTHER PEAKS ARE AFFECTED BY THE BALLISTIC DEFICIT ERROR, BUT THE SIMPLIFIED ASSUMPTION IS PRACTICALLY TRUE FOR ALL CASES. DPLMS METHOD ACHIEVES THE BEST PERFORMANCE

	Energy resolution [keV]		
Method	*Electronic line*	*1.17 MeV line*	*1.33 MeV line*
Deconvolution	4.72 ± 0.03	4.69 ± 0.03	4.63 ± 0.03
Fourier (delta-like input)	5.94 ± 0.04	5.86 ± 0.04	5.64 ± 0.04
Fourier (more correct input)	4.88 ± 0.03	4.71 ± 0.03	4.57 ± 0.03
DPLMS	**4.67 ± 0.03**	**4.60 ± 0.03**	**4.58 ± 0.03**

as the DPLMS method works on the experimental pulses and does not require any modeling of the signal at all, it can synthesize a WF with a more precise flat top and a slightly better energy resolution (see Fig. 5 and Table I).

V. CONCLUSIONS

The experimental comparison of different methods to calculate optimum filters with arbitrary constraints in the presence of any noise power spectrum has been presented.

Three methods have been evaluated, which are among the state-of-the-art of digital filter-synthesis techniques for shaped signals: a classic deconvolution method; the Fourier method; and the digital penalized least mean squares method.

All methods have been implemented on a personal computer. As it only takes a few seconds to synthesize the filters and to calculate the corresponding ENC^2, it is quite easy to explore the loss in ENC^2 due to the imposed constraints, and find the best tradeoff between the energy resolution and the fulfillment of the additional requirements.

Online digital spectroscopy is nowadays usually carried out by means of standard electronic devices (field programmable gate array and digital signal processor). Although no dedicated hardware has been used for this experiment, all the digital filters under comparison can be implemented by means of reconfigurable pulse-processing setups (e.g., those using customized FIR filters). If such a hardware device is available, then all the digital filters can be easily implemented. In case of a hardware device with limited filtering capabilities, the pulse-processing technique based on the moving window deconvolution may be preferable, because it is computationally simpler.

As a first point, this experimental comparison clearly shows that the synthesis of optimum filters for real measurement setups is a difficult task. If *a priori* assumptions are made during filter synthesis, there is a high chance that the synthesized "optimum" filter will have much poorer performance than standard filters. On the contrary, the DPLMS method does not require any *a priori* assumption, and is then a practical and effective tool for optimum filter synthesis.

As a second point, these results show that in order to precisely estimate, from the quantitative viewpoint, how much gain can be achieved by using optimum filters synthesized by the DPLMS method instead of standard filtering techniques, further experiments have to be done. Specifically, in order to reduce the uncertainty in the estimation of the energy resolution, a much higher number of pulses will have to be processed. While at the time of writing, this is not reasonable because of the relative inefficiency in the data collection process, we plan to achieve it in the near future, either by means of a faster data collection setup, or using a custom analog-to-digital acquisition board with DSP capabilities that we are going to develop.

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